Gravity vs. Quantum theory: Is electron really pointlike?

Alexander Burinskii

Lab. of Theor. Phys. , NSI, Russian Academy of Sciences, B. Tulskaya 52 Moscow 115191 Russia

E-mail: bur@ibrae.ac.ru

Abstract. The observable gravitational and electromagnetic parameters of an electron: mass m, spin $J = \hbar/2$, charge e and magnetic moment $ea = e\hbar/(2m)$ indicate unambiguously that the electron should had the Kerr-Newman background geometry – exact solution of the Einstein-Maxwell gravity for a charged and rotating black hole. Contrary to the widespread opinion that gravity plays essential role only on the Planck scales, the Kerr-Newman gravity displays a new dimensional parameter $a = \hbar/(2m)$, which for parameters of an electron corresponds to the Compton wavelength and turns out to be very far from the Planck scale. Extremely large spin of the electron with respect to its mass produces the Kerr geometry without horizon, which displays very essential topological changes at the Compton distance resulting in a two-fold structure of the electron background. The corresponding gravitational and electromagnetic fields of the electron are concentrated near the Kerr ring, forming a sort of a closed string, structure of which is close to the described by Sen heterotic string. The indicated by Gravity stringlike structure of the electron contradicts to the statements of Quantum theory that electron is pointlike and structureless. However, it confirms the peculiar role of the Compton zone of the "dressed" electron and matches with the known limit of the localization of the Dirac electron. We discuss the relation of the Kerr string with the low energy string theory and with the Dirac theory of electron and suggest that the predicted by the Kerr-Newman gravity closed string in the core of the electron, should be experimentally observable by the novel regime of the high energy scattering – the Deeply Virtual (or "nonforward") Compton Scattering".

1. Introduction

Modern physics is based on Quantum theory and Gravity. The both theories are confirmed experimentally with great precision. Nevertheless, they are conflicting and cannot be unified in a whole theory. General covariance is main merit of General Relativity and the main reason of misinterpretation. The freedom of coordinate transformations is one of the source of the conflict. One of the source of the problems is the absence of the usual plane waves in general relativity, which causes the conflict with the Fourier transform and prevents expansion of the quantum methods to the curved spacetimes.

The analogs of the plane waves in gravity are the pp-wave solutions which are singular, either at infinity or at some lightlike (twistor) line, forming a singular ray similar to the laser beam [1]. The pp-wave singular beams are modulated by the usual plane waves and form singular strings, which are in fact the fundamental strings of the low energy string theory. Moreover, it turns out that the pp-waves don't admit α' stringy corrections [1, 2, 3], and therefore they are exact solutions to the full string theory.

The null Killing direction of the pp-waves, k_μ, $\quad (k_\mu k^\mu = 0)$, is adapted to the Kerr-Schild (KS) form of the metric $g^{\mu\nu} = \eta^{\mu\nu} + 2Hk^\mu k^\nu$ which is rigidly related with the auxiliary Minkowski background geometry $\eta^{\mu\nu}$. The KS class of metrics is matched with the light-cone structure of the Minkowski background which softens conflict with quantum theory. In spice of the extreme rigidity, the Kerr-Schild coordinate system allows one to describe practically all the physically interesting solutions of General Relativity, for example:

- rotating black holes and the sources without horizon,
- de Sitter and Anti de Sitter spaces, and their rotating analogues,
- combinations of a black hole inside the de Sitter or AdS background spacetime,
- the opposite combinations: dS or AdS spaces as regulators of the black hole geometry
- charged black holes and rotating stars, and so on.

In particular, the Kerr-Schild metric describes the Kerr-Newman (KN) solution for a charged and rotating black hole, which for the case of very large angular momentum may be considered as a model of spinning particles.

In 1968 Carter obtained, [4, 5], that the KN solution has the gyromagnetic ratio $g = 2$ as that of the Dirac electron, which initiated a series of the works on the KN electron model [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

In this paper we discuss one of the principal contradictions between Quantum theory and Gravity, the question on the shape and size of the electron, believing that the nature of the electron is principal point for understanding of Quantum Theory. Quantum theory states that electron is pointlike and structureless. In particular, Frank Wilczek writes in [20]: "...There's no evidence that electrons have internal structure (and a lot of evidence against it)", while the superstring theorist Leonard Susskind notes that electron radius is "...most probably not much bigger and not much smaller than the Planck length..", [21]. This point of view is supported by the experiments with high energy scattering, which have not found the electron structure down to $10^{-16}cm$.

On the other hand, the experimentally observable parameters of the electron: angular momentum J, mass m, charge e and the magnetic moment μ indicate unambiguously that its background geometry should be very close to the corresponding Kerr-Newman (KN) solution of the Einstein-Maxwell field equations. The observed parameters of the electron J, m, e, μ determine also the corresponding parameters of the KN solution: the mass m, charge e, and the new dimensional parameter $a = L/m$ which fixes radius of the Kerr singular ring. The fourth observable parameter of the electron, magnetic momentum μ, conform to the KN solution automatically as consequence of the above discussed specific gyromagnetic ratio of the Dirac electron coinciding with that of the KN solution. As a result, the KN solution indicates a characteristic radius of the electron as the Compton one $a = \hbar/(2m)$, corresponding to the radius of the Kerr singular ring. Therefore, contrary to Quantum theory, the KN gravity predicts the ring like structure of the electron and its Compton size.

The metric of the Kerr-Newman spacetime has the form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \tag{1}$$

where

$$H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}, \tag{2}$$

and the electromagnetic (EM) vector potential of the KN solution is

$$\alpha^\mu_{KN} = Re\frac{e}{r + ia\cos\theta}k^\mu, \tag{3}$$

where r and θ are Kerr's oblate spheroidal coordinates which are related to the Cartesian coordinates as follows

$$\begin{aligned} x + iy &= (r + ia)e^{i\phi} \sin\theta \\ z &= r \cos\theta. \end{aligned} \tag{4}$$

The metric and EM field are aligned with the null vector field k^μ forming a Principal Null Congruence (the 'Kerr congruence'), see Figure.1. The Kerr congruence is determined by *the Kerr Theorem* in twistor terms (each line of the congruence is a twistor null line). Although the KN spacetime is curved, the Kerr congruence foliates it into a family of the flat complex twistor null planes, which allows one to use in the Kerr-Schild spaces a twistor version of the Fourier transform, which forms a holographic bridge between the classical Kerr-Schild gravity and Quantum theory, [22, 23].

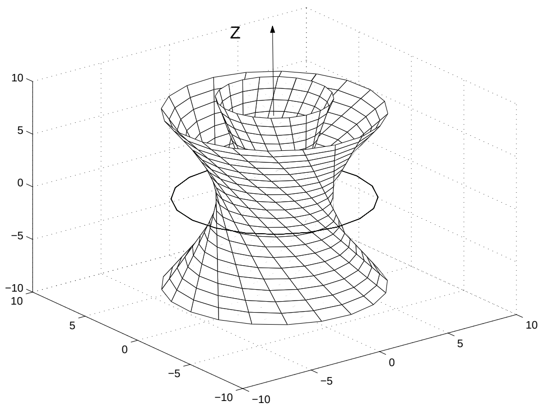

Figure 1. Vortex of the Kerr congruence. Twistor null lines are focused on the Kerr singular ring, forming a circular gravitational waveguide, or string with lightlike excitations.

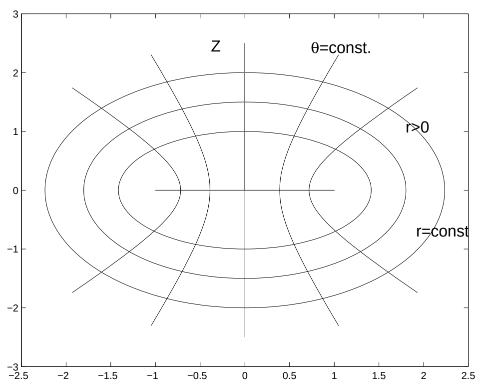

Figure 2. Oblate coordinate system r, θ with focal points at $r = \cos\theta = 0$ forms a twofold analytic covering: for $r > 0$ and $r < 0$.

The KN gravitational and EM fields are concentrated near the Kerr singular ring, which appears in the rotating BH solutions instead of the pointlike Schwarzschild singularity. One sees that radius of the ring $a = J/m$ increases for the small masses and is proportional to the spin J. Therefore, contrary to the characteristic radius of the Schwarzschild solution (related with position of the BH horizon, $r_g = 2m$), the characteristic extension of the KN gravitational field turns out to be much beyond the Planck length, and corresponds to the Compton length, $r_{compt} = a = \hbar/(2m)$, or to the radius of a "dressed" electron. In the units $c = \hbar = G = 1$, mass of the electron is $m \approx 10^{-22}$, while $a = J/m \approx 10^{22}$. Therefore, $a >> m$, and the black hole horizons disappear, showing that the Kerr singular ring is naked. In this case the Kerr spacetime turns out to be twosheeted, since the Kerr ring forms its branch line creating a twosheeted topology. The relations (1) and (3) show that the gravitational and electromagnetic fields of the KN solution are concentrated in a thin vicinity of the Kerr singular ring $r = \cos\theta = 0$, forming a type of "gravitational waveguide", or a closed string, [12]. The Kerr string takes the Compton radius, corresponding to the size of a "dressed" electron in QED and to the limit of localization of the electron in the Dirac theory [24].

There appear two questions:

(A) How does the KN gravity know about one of the principal parameters of Quantum theory? and

(B) Why does Quantum theory works successfully on the flat spacetime, ignoring the stringy defect of the background geometry?

A small and slowly varying gravitational field could be neglected, however the stringlike KN singularity forms a branch-line of the KS spacetime, and such a topological defect cannot be ignored. A natural resolution of this trouble could be the assumption that there is an underlying theory providing the consistency of quantum theory and gravity. In this paper we suggest a rather unexpected resolution of this puzzle, claiming that underlying theory is the low energy string theory, in which the closed string is created by the KN gravity related with twistorial structure of the Kerr-Schild pre-quantum geometry [22, 23]. The Kerr singular ring is generated as a caustic of the Kerr twistor congruence and forms a closed string on the boundary of the Compton area of the electron. The KN gravity indicates that this string should represent a principal element of the extended electron structure.

If the closed Kerr string is really formed on the boundary of the Compton area, it should be experimentally observable. There appears the question while it was not obtained earlier in the high energy scattering experiments. We find some explanation to this fact and arrive at the conclusion that the KN string should apparently be detected by the novel experimental regime of the high energy scattering which is based on the theory of Generalized Parton Distributions (GPD), and corresponds to a "non-forward Compton scattering" [25, 26], suggested recently for tomography of the particle images [27].

2. Twosheetedness of the Kerr-Geometry

In the KS representation [5], a few coordinate systems are used simultaneously. In particular, *the null Cartesian coordinates*

$$\zeta = (x+iy)/\sqrt{2},\ \bar\zeta = (x-iy)/\sqrt{2},\ u = (z-t)/\sqrt{2},\ v = (z+t)/\sqrt{2}$$

are used for description of the Kerr congruence in the differential form

$$k_\mu dx^\mu = P^{-1}(du + \bar Y d\zeta + Y d\bar\zeta - Y\bar Y dv), \tag{5}$$

via the complex function $Y(x) = e^{i\phi}\tan\frac{\theta}{2}$, which is a projective angular coordinate on the celestial sphere,

$$Y(x) = e^{i\phi}\tan\frac{\theta}{2}. \tag{6}$$

2.1. The Kerr Theorem

Kerr congruence (PNC) is controlled by *THE KERR THEOREM:*

The geodesic and shear-free Principal null congruences (type D metrics) are determined by holomorphic function $Y(x)$ which is analytic solution of the equation

$$F(T^a) = 0\ , \tag{7}$$

where F is an arbitrary analytic function of the projective twistor coordinates

$$T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar\zeta\}. \tag{8}$$

The Kerr theorem is a practical tool for the obtaining the exact Kerr-Schild solutions. The following sequence of steps is assumed:

$$F(T^a) = 0 \Rightarrow F(Y, x^\mu) = 0 \Rightarrow Y(x^\mu) \Rightarrow k^\mu(x) \Rightarrow g^{\mu\nu}$$

For the Kerr-Newman solution function F is quadratic in Y, which yields TWO roots $Y^{\pm}(x)$ corresponding to two congruences!

As a result the obtained two congruences (IN and OUT) determine two sheets of the Kerr solution: the “negative (−)” and “positive (+)” sheet, where the fields change their directions. In particular, two different congruences $k^{\mu(+)} \neq k^{\mu(-)}$ determine two different KS metrics $g^{(+)}_{\mu\nu} \neq g^{(-)}_{\mu\nu}$ on the same Minkowski background. As it shows the Fig.1, the Kerr congruence propagates analytically from IN to OUT- sheet via the disk $r = 0$, and therefore, the two KS sheets are linked analytically. The twosheeted KN space is parametrized by the oblate spheroidal coordinate system $r, \ \theta, \phi$, which tends asymptotically, by $r \to \infty$, to the usual spherical coordinate system. Twosheetedness is the long-term mystery of the Kerr solution! For the multiparticle Kerr-Schild (KS) solutions, [34], the Kerr theorem yields many roots $Y^i, \ i = 1, 2, ...$ of the Kerr equation $F(Y) = 0$, and the KS geometry turns out to be *multivalued* and *multisheeted*.

The extremely simple form of the Kerr-Schild metric (1) is related with complicate form of the Kerr congruence, which represents a type of deformed (twisted) hedgehog. In the rotating BH solutions the usual pointlike singularity inside the BH turns into a *a closed singular ring,* which is interpreted as a closed string in the corresponding models of the spinning elementary particles [15]. The KN twosheetedness was principal puzzle of the Kerr geometry for four decades and determined development of the KN electron models along two principal lines of investigation: I) the bubble models, and II) the stringlike models.

I. – In 1968 Israel suggested to truncate negative KN sheet, $r < 0$,, and replace it by the *rotating disklike source* ($r = 0$) spanned by the Kerr singular ring of the Compton radius $a = \hbar/2m$, [9]. Then, Hamity obtained in [28] that the disk has to be rigidly rotating, which led to a reasonable interpretation of the matter of the source as an exotic stuff with zero energy density and negative pressure. The matter distribution appeared singular at the disk boundary, forming an additional closed string source, and López suggested in [13] to regularize this source, covering the Kerr singular ring by a disklike ellipsoidal surface. As a result, the KN source was turned into a rotating and charged oblate bubble with a flat interior, and further it was realized as a regular soliton-like bubble model [6], in which the boundary of the bubble is formed by a domain wall interpolating between the external KN solution and a flat pseudovacuum state inside the bubble.

II. The stringlike models of the KN source retain the twosheeted topology of the KN solution, forming a closed 'Alice' string of the Compton size [12, 8, 29]. The Kerr singular ring is considered as a waveguide for electromagnetic traveling waves generating the spin and mass of the KN solution in accordance with the old Wheeler's "geon" model of ‘mass without mass’ [30, 10, 11, 31].[1] In this paper we concentrate on the stringlike model of the electron, which displays close relations to the low energy string theory. The bubble model of regularization of the KN solution is discussed briefly in sec.5 along our previous papers.

3. The Kerr singular ring as a closed string

Exact *non-stationary* solutions for electromagnetic excitations on the Kerr-Schild background, [29, 33, 22], showed that there are no smooth harmonic solutions. The typical exact electromagnetic solutions on the KN background take the form of singular beams propagating along the rays of PNC, contrary to smooth angular dependence of the wave solutions used in perturbative approach!

Position of the horizon for the excited KS black holes solutions is determined by function H

[1] Note also that the KN twosheetedness represents a natural realization of the Einstein-Rosen bridge model related with the “in-going“ and “out-going” radiation [33], as well as with the Wheeler “charge without charge” model.

which has for the exact KS solutions the form, [5],

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2 \cos^2\theta} \ , \tag{9}$$

where $\psi(x)$ is related to the vector potential of the electromagnetic field

$$\alpha = \alpha_\mu dx^\mu = -\frac{1}{2} Re \ [(\frac{\psi}{r + ia\cos\theta})e^3 + \chi d\bar{Y}], \quad \chi = 2\int (1 + Y\bar{Y})^{-2}\psi dY \ , \tag{10}$$

which obeys the alignment condition

$$\alpha_\mu k^\mu = 0. \tag{11}$$

The equations (1)and (9) display compliance and elasticity of the horizon with respect to the electromagnetic field.

The Kerr-Newman solution corresponds to $\psi = q = const.$. However, any nonconstant holomorphic function $\psi(Y)$ yields also an exact KS solution, [5]. On the other hand, any nonconstant holomorphic functions on sphere acquire at least one pole. A single pole at $Y = Y_i$

$$\psi_i(Y) = q_i/(Y - Y_i) \tag{12}$$

produces the beam in angular directions

$$Y_i = e^{i\phi_i} \tan\frac{\theta_i}{2}. \tag{13}$$

The function $\psi(Y)$ acts immediately on the function H which determines the metric and the position of the horizon. The analysis showed, [33], that electromagnetic beams have very strong back reaction to metric and deform topologically the horizon, forming the holes which allows matter to escape interior (see fig.3).

The exact KS solutions may have arbitrary number of beams in different angular directions $Y_i = e^{i\phi_i} \tan\frac{\theta_i}{2}$. The corresponding function

$$\psi(Y) = \sum_i \frac{q_i}{Y - Y_i}, \tag{14}$$

leads to the horizon with many holes. In the far zone the beams tend to the known exact singular pp-wave solutions. The considered in [34] multi-center KS solutions showed that the beams are extended up to the other matter sources, which may also be assumed at infinity.

The stationary KS beamlike solutions may be generalized to the time-dependent wave pulses, [29], which tend to exact solutions in the low-frequency limit.

Since the horizon is extra sensitive to electromagnetic excitations, it may also be sensitive to the vacuum electromagnetic field which is exhibited classically as a Casimir effect, and it was proposed in [22] that the vacuum beam pulses shall produce a fine-grained structure of fluctuating microholes in the horizon, allowing radiation to escape interior of black-hole, as it is depicted on Fig.3.

The function $\psi(Y, \tau)$, corresponding to beam pulses, has to depend on retarded time τ and satisfy to the obtained in [5] nonstationary Debney-Kerr-Schild (DKS) equations leading to the extra long-range radiative term $\gamma(Y, \tau)Z$. The expression for the null electromagnetic radiation take the form, [5], $F^{\mu\nu} = Re\mathcal{F}_{31} e^{3\mu} \wedge e^{1\nu}$, where

$$\mathcal{F}_{31} = \gamma Z - (AZ)_{,1} \ , \tag{15}$$

$Z = P/(r + ia\cos\theta)$, $P = 2^{-1/2}(1 + \bar{Y}\bar{\bar{Y}})$, and the null tetrad vectors have the form $e^{3\mu} = Pk^\mu$, $e^{1\mu} = \partial_{\bar{Y}} e^{3\mu}$.

The long-term attack on the DKS equations has led to the obtained in [22] time-dependent solutions which revealed a holographic structure of the fluctuating Kerr-Schild spacetimes and showed explicitly that the electromagnetic radiation from a black-hole interacting with vacuum contains two components:

a) a set of the singular beam pulses (determined by function $\psi(Y,\tau)$,) propagating along the Kerr PNC and breaking the topology and stability of the horizon;

b) the regularized radiative component (determined by $\gamma_{reg}(Y,\tau)$) which is smooth and, similar to that of the the Vaidya 'shining star' solution, determines evaporation of the black-hole,

$$\dot{m} = -\frac{1}{2}P^2 < \gamma_{reg}\bar{\gamma}_{reg} > . \tag{16}$$

The mysterious twosheetedness of the KS geometry plays principal role in the holographic black-hole spacetime [22], allowing one to consider action of the electromagnetic in-going vacuum as a time-dependent process of scattering. The obtained solutions describe excitations of electromagnetic beams on the KS background, the fine-grained fluctuations of the black-hole horizon, and the consistent back reaction of the beams to metric [33, 22]. The holographic space-time is twosheeted and forms a fluctuating pre-geometry which reflects the dynamics of the singular beam pulses. This pre-geometry is classical, but has to be still regularized to get the usual smooth classical space-time. In this sense, it takes an intermediate position between the classical and quantum gravity.

Meanwhile, the function $Z = P/(r + ia\cos\theta)$ tends to infinity near the Kerr ring, indicating that any electromagnetic excitation of the KN geometry should generates the related singular traveling waves along the Kerr singular ring, and therefore, the 'axial' singular beams turn out to be topologically coupled with the 'circular' traveling waves, see Fig.4. The both these excitations travel at the speed of light, and the 'axial' beams tend asymptotically (by $r \to \infty$) to the pp-wave (plane fronted wave) solutions, for which the vector k_μ in (1) forms a covariantly constant Killing direction.

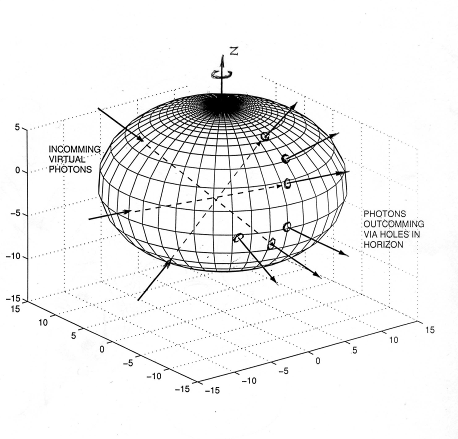

Figure 3. Excitations of a black hole by a weak EM field creates a series of fluctuating twistor-beams (outgoing pp-waves) which perforate the black hole horizon, covering it by the fluctuating micro-holes.

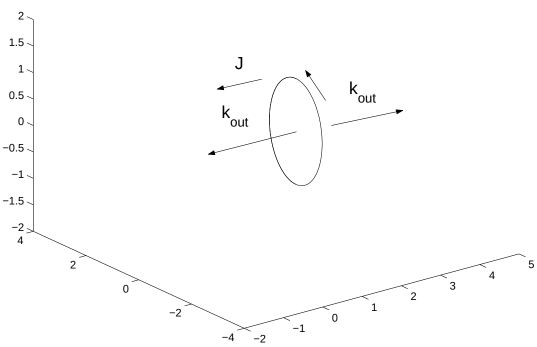

Figure 4. Skeleton of the Kerr geometry [29, 22] formed by the topologically coupled 'circular' and 'axial' strings.

The pp-waves take very important role in superstring theory, forming the singular classical solutions to the low-energy string theory [1]. The string solutions are compactified to four dimensions and the singular pp-waves are regarded as the massless fields around a lightlike fundamental string. It is suspected that the singular source of the string will be smoothed out in the full string theory, taking into account all orders in α'. In the nonperturbative approach based on analogues between the strings and solitons, the pp-wave solutions are considered as fundamental strings [35]. The pp-waves may carry traveling electromagnetic and gravitational waves which represents propagating modes of the fundamental string [36]. In particular, the generalized pp-waves represent the singular strings with traveling electromagnetic waves [29, 2]. It has been noticed that the field structure of the Kerr singular ring is similar to a closed pp-wave string [11, 12]. This similarity is not incidental, since many solutions to the Einstein-Maxwell theory turn out to be particular solutions to the low energy string theory with a zero (or constant) axion and dilaton fields. Indeed, the bosonic part of the action for the low-energy string theory takes after compactification to four dimensions the following form, [37],

$$S = \int d^4x\sqrt{-g}(R - 2(\partial\phi)^2 - e^{-2\phi}F^2 - \frac{1}{2}e^{4\phi}(\partial a)^2 - aF\tilde{F}), \tag{17}$$

which contains the usual Einstein term $S_g = \int d^4x\sqrt{-g}R$ completed by the kinetic term for dilaton field $-2(\partial\phi)^2$ and by the scaled by ϕ electromagnetic field. The last two terms are related with axion field a and represent its nonlinear coupling with dilaton field $-\frac{1}{2}e^{4\phi}(\partial a)^2$ and interaction of the axion with the dual electromagnetic field $\tilde{F}_{\mu\nu} = \epsilon^{\lambda\rho}_{\mu\nu}F_{\lambda\rho}$.

It follows immediately that *any solution of the Einstein gravity, and in particular the Kerr solution, is to be exact solution of the effective low energy string theory* with a zero (or constant) axion and dilaton fields. Situation turns out to be more intricate for the Einstein-Maxwell solutions since the electromagnetic invariant F^2 plays the role of the source of dilaton field. Similarly, the term $F\tilde{F}$ turns out to be the source of the axion field. The stringy analog to the Kerr-Newman solution with nontrivial axion and dilaton fields was obtained by Sen [38], and it was shown in [15], that the field around the singular string in the 'axidilatonic' Kerr-Sen solution is very similar to the field around a heterotic string.[2] This proximity of the pure gravitational strings [12] to the low-energy string theory allows us to consider the Kerr singular ring as a closed heterotic string and the corresponding traveling waves as its lightlike propagating modes. The axidilaton field is related with the string tension, and therefore, the nontrivial solutions to the low energy string theory should be very important, allowing one to estimate the mass-energy of the excited string states. The structure of the Lagrangian (17) shows that the axion field involves the dual magnetic field, and therefore, the complex axidilaton combination may generate the duality rotation and create an additional twist of the electromagnetic traveling waves. However, the exact solutions of this type are so far unknown. Note also that the axidilaton field appears naturally in the based on the special-Kähler geometry 4D models of black holes in supergravity, which may have important consequences for the models of regularized KN solution [6].

Assuming that the lightlike string forms a core of the electron structure, we have to obtain a bridge to the one-particle quantum theory. Traveling waves along the KN closed string generate the spin and mass of the stringlike particle. Physically, it is equivalent to the original Wheeler's model of 'mass without mass' [30]. In the next section we show emergence of the Dirac equation from this physical picture.

4. Mass without mass

The puzzle of "zitterbewegung" and the known processes of annihilation of the electron-positron pairs brought author in 1971 to the Wheeler "geon" model of the "mass without mass" [30].

[2] Peculiarity of the heterotic strings is related with the lightlike current and the lightlike bosonic traveling modes of one direction.

In [31] we considered a massless particle circulating around z-axis. Its local 4-momentum is lightlike,

$$p_x^2 + p_y^2 + p_z^2 = E^2, \tag{18}$$

while the effective mass-energy was created by an averaged orbital motion,

$$< p_x^2 > + < p_y^2 > = \tilde{m}^2. \tag{19}$$

Averaging (18) under the condition (19) yields

$$< p_x^2 + p_y^2 + p_z^2 > = \tilde{m}^2 + p_z^2 = E^2. \tag{20}$$

Quantum analog of this model corresponds to a wave function $\psi(\vec{x}, t)$ and operators, $\vec{p} \to \hat{\vec{p}} = -i\hbar\nabla, \quad \hat{E} = i\hbar\partial_t$. From (18) and (19) we obtain the D'Alembert equation $\partial^\mu \partial_\mu \psi = 0$ and the constraint $(\partial_x^2 + \partial_y^2)\psi = 0$, which for the chosen coordinate system are reduced to the equations

$$(\partial_x^2 + \partial_y^2)\psi = \tilde{m}^2\psi = (\partial_t^2 - \partial_z^2)\psi, \tag{21}$$

and may be separated by the ansatz

$$\psi = \mathcal{M}(x, y)\Psi_0(z, t). \tag{22}$$

The RHS of (21) yields the usual equation for a massive particle, $(\partial_t^2 - \partial_z^2)\Psi_0 = \tilde{m}^2\Psi_0$, and the corresponding (de Broglie) plane wave solution

$$\Psi_0(z, t) = \exp \frac{i}{\hbar}(zp_z - Et), \tag{23}$$

while the l.h.s. determines the "internal" structure factor

$$\mathcal{M}_\nu = \mathcal{H}_\nu(\frac{\tilde{m}}{\hbar}\rho) \exp\{i\nu\phi\}, \tag{24}$$

in polar coordinates ρ, ϕ, where $\mathcal{H}_\nu(\frac{\tilde{m}}{\hbar}\rho)$ are the Hankel functions of index ν. $\mathcal{M}_\nu$ are eigenfunctions of operator $\hat{J}_z = \frac{\hbar}{i}\partial_\phi$ with eigenvalues $J_z = \nu\hbar$. For electron we have $J_z = \pm\hbar/2, \quad \nu = \pm 1/2$, and the factor

$$\mathcal{M}_{\pm 1/2} = \rho^{-1/2} \exp\{i(\frac{\tilde{m}}{\hbar}\rho \pm \frac{1}{2}\phi)\} \tag{25}$$

creates a singular ray along z-axis, which forms a branch line, and the wave function is twovalued.

There are diverse generalizations of this solution. First of all, there may be obtained the corresponding wave functions based on the eigenfunctions of the operator of the total angular momentum and simultaneously of the spin projection operator. Next, the treatment may be considered in a Lorentz covariant form for arbitrarily positioned and oriented wave functions. And finally, the corresponding spinor models, together with all the corresponding spinor solutions may also be obtained (see [32]).

Principal peculiarity of the obtained massless model is that the usual plane wave functions are replaced by the vortex waves generating the spin and mass of the particle-like solutions of the *massless* equations, and therefore, the contradiction between the massive wave equation and the lightlike zitterbewegung (determined by the Dirac operators α) disappears. On the other hand, the wave functions (22) are factorized into the usual plane waves $\Psi_0(z, t)$ and the string-like singular factors $\mathcal{M}(x, y)$ playing the role of singular carriers of de Broglie waves, which reproduces de Broglie's wave-pilot conception, which is however principally different from the corresponding Bohm model.

It should also be mentioned that the characteristic spinor twovaluedness appears also for scalar waves, as a consequence of the topological twosheetedness generated by the singular branch line. In the Kerr geometry this 'axial' branch line is linked with the Kerr 'circular' branch line (Figure 1.), forming a topologically nontrivial spacetime structure of the KN geometry, (Figure 2.).

5. Regularization: Electron as a gravitating soliton

The experimentally indicated KN background of the electron exhibits the closed singular string which contradicts to the Quantum assumptions that the gravity is negligible and the background is flat. As a result, the justification of the Dirac electron theory and QED requires *regularization of the KN metric,* which should be performed with invariability of its asymptotic form. Similar regularization of the singular strings of the low-energy string theory is assumed in the full string theory [39]. For the KN solution this problem is close related with general problem of the regularization of the black hole singularity [40] and with the old problem of the regular source of the KN solution [16, 41, 42].

5.1. Gravitational aspect

The used by Israel truncation of the negative sheet of the KN solution [9] led to the disklike model of the KN source which retained the Kerr singular ring. It was replaced by López by the regular model of a rigidly rotating charged bubble with a flat interior [13], which was a prototype of the gravitating soliton model [6]. The singular region of the KN solution is rejected in the López model and replaced by the flat space-time, forming a bubble with flat interior. One should retain the asymptotic form of the external KN solution and provide a smooth matching of the external metric with the flat bubble interior. It is achieved by the special chose of the bubble boundary r_b which is determined by the condition $H(r_b) = 0$. From (2) one obtains

$$r_b = r_e = e^2/(2m), \tag{26}$$

where r_b is the Kerr ellipsoidal radial coordinate, (4). As a result, the regular KN source takes the form of an oblate disk of the Compton radius $r_c \approx a = \hbar/(2m)$ with the thickness $r_e = e^2/(2m)$ corresponding to the known 'classical size' of the electron. One sees that the consistent regularization needs an extension up to the Compton distance. The electromagnetic field is also regularized, and the López bubble model represents a charged and rotating singular shell. The corresponding smooth and regular rotating sources of the Kerr-Schild class were considered in [16, 41] on the base of the generalized KS class of metrics suggested by Gr̈ses and Gürsey in [42]. The function H in the generalized KS form of metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu,$$

is taken to be

$$H = f(r)/(r^2 + a^2\cos^2\theta), \tag{27}$$

where the function $f(r)$ interpolates between the inner regular metric and the external KN solution. By such a deformation, the Kerr congruence, determined by the vector field $k^\mu(m) \in M^4$, should retain the usual KS form (5).

It allows one to suppress the Kerr singular ring ($r = \cos\theta = 0$) by a special choice of the function $f(r)$.

The regularized solutions have tree regions:

i) the Kerr-Newman exterior, $r > r_0$, where $f(r) = mr - e^2/2$,

ii) interior $r < r_0 - \delta$, where $f(r) = f_{int}$ and function $f_{int} = \alpha r^n$, and $n \geq 4$ to suppress the singularity at $r = 0$, and provide the smoothness of the metric up to the second derivatives.

iii) intermediate region providing a smooth interpolation between i) and ii).

5.2. Material aspect

To remove the Kerr-Newman singularity, one has to set for the internal region

$$f_{int} = \alpha r^4.$$

In this case, the Kerr singularity is replaced by a regular rotating internal space-time with a constant curvature, $R = -24\alpha$ [16, 41].

The functions

$$D = -\frac{f''}{\Sigma}, \quad G = \frac{f'r - f}{\Sigma^2}. \tag{28}$$

determine stress-energy tensor in the orthonormal tetrad $\{u, l, m, n\}$ connected with the Boyer-Lindquist coordinates,

$$T_{ik} = (8\pi)^{-1}[(D + 2G)g_{ik} - (D + 4G)(l_i l_k - u_i u_k)]. \tag{29}$$

In the above formula, u^i is a timelike vector field given by

$$u^i = \frac{1}{\sqrt{\Delta\Sigma}}(r^2 + a^2, 0, 0, a).$$

This expression shows that the matter of the source is separated into ellipsoidal layers corresponding to constant values of the coordinate r, each layer rotates with angular velocity $\omega(r) = \frac{u^\phi}{u^0} = a/(a^2 + r^2)$. This rotation becomes rigid only in the thin shell approximation $r = r_0$. The linear velocity of the matter w.r.t. the auxiliary Minkowski space is $v = \frac{a \sin\theta}{\sqrt{a^2+r^2}}$, so that on the equatorial plane $\theta = \pi/2$, for small values of r ($r \ll a$), one has $v \approx c = 1$, that corresponds to an oblate, relativistically rotating disk.

The energy density ρ of the material satisfies to $T^i_k u^k = -\rho u^i$ and is, therefore, given by

$$\rho = \frac{1}{8\pi} 2G. \tag{30}$$

Two distinct spacelike eigenvalues, corresponding to the radial and tangential pressures of the non rotating case are

$$p_{rad} = -\frac{1}{8\pi} 2G = -\rho, \tag{31}$$

$$p_{tan} = \frac{1}{8\pi}(D + 2G) = \rho + \frac{D}{8\pi}. \tag{32}$$

In the exterior region function f must coincide with Kerr-Newman solution, $f_{KN} = mr - e^2/2$.

There appears a transition region placed in between the boundary of the matter object and the de Sitter core. This transition region has to be described by a smooth function $f(r)$ which interpolates between the functions $f_{int}(r)$ and $f_{KN}(r)$. Graphical analysis allows one to determine position of the bubble boundary r_b, [41, 43]. For the López model of the flat interior $f_{int} = 0$ and $r_b = r_e = e^2/(2m)$. The case $\alpha > 0$ corresponds to de Sitter interior and uncharged source. There is only one intersection between $f_{int}(r) = \alpha r^4$ and $f_{KN}(r) = mr$. The position of the transition layer will be $r_b = (m/\alpha)^{-1/3}$. The second derivative of the corresponding interpolating function will be negative at this point, yielding an extra contribution to the positive tangential pressure in the transition region.

5.3. Chiral field model and the Higgs field

In accordance with the Einstein equations, the considered smooth and regular metric should be generated by a system of the matter fields forming a classical source of the vacuum bubble. In the suggested in [6] soliton model, the smooth phase transition from the external KN solution to the internal 'pseudovacuum state' is generated by a supersymmetric set of chiral fields $\Phi^i, \quad i = 1, 2, 3$, [6, 16, 44] controlled by the suggested by Morris [45] super-potential

$$W = \lambda Z(\Sigma\bar{\Sigma} - \eta^2) + (cZ + \mu)\Phi\bar{\Phi}, \tag{33}$$

where $c,\ \mu,\ \eta,\ \lambda$ are the real constants, and we have set $\Phi^1 = \Phi,\ \Phi^2 = Z$ and $\Phi^3 = \Sigma$. The potential is determined by the usual relations of the supersymmetric field theory [46]

$$V(r) = \sum_i |\partial_i W|^2, \tag{34}$$

where $\partial_1 = \partial_\Phi,\ \partial_2 = \partial_Z,\ \partial_3 = \partial_\Sigma$. The vacuum states are determined by the conditions $\partial_i W = 0$ which yield $V = 0$

i) for 'false' vacuum ($r < r_0$): $Z = -\mu/c; \Sigma = 0; |\Phi| = \eta\sqrt{\lambda/c}$, and also

ii) for 'true' vacuum ($r > r_0$) : $Z = 0; \Phi = 0; \Sigma = \eta$.

which provides a phase transition from the external KN 'vacuum state', $V_{ext} = 0$, to a flat internal 'pseudovacuum' state, $V_{int} = 0$, providing regularization of the Kerr singular ring, [16, 44, 6]. One of the chiral fields, Φ^1, is set as the Higgs field $\Phi^1 \equiv \Phi = \Phi_0 \exp(i\chi)$. As a result of the phase transition, the Higgs field $\Phi_0 \exp(i\chi)$ with a nonzero vev Φ_0 and the phase χ fills interior of the bubble and regularizes the electromagnetic Kerr-Newman field by the Higgs mechanism of broken symmetry.

5.4. Regularization of the electromagnetic KN field

The electromagnetic KN field inside the bubble interacts with the Higgs field in agreement with Landau-Ginzburg type field model with the Lagragian

$$\mathcal{L}_{NO} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\mathcal{D}_\mu\Phi)(\mathcal{D}^\mu\Phi)^* + V(r), \tag{35}$$

where $\mathcal{D}_\mu = \nabla_\mu + ie\alpha_\mu$ are to be covariant derivatives. This model was used by Nielsen and Olesen [47] for the obtaining the string-like solutions in superconductivity. The model determines the current

$$I_\mu = \frac{1}{2}e|\Phi|^2(\chi,_\mu + e\alpha_\mu) \tag{36}$$

as a source of the Maxwell equations $F^{\nu\mu}{}_{;\nu} = I^\mu$. This current should vanish inside the bubble, which sets a relation between incursion of the phase of the Higgs field and the value of the vector potential of the KN solution on the boundary of the bubble

$$\chi,_\mu = -e\alpha_\mu^{(str)}. \tag{37}$$

The maximal value of the regularized vector potential $\alpha_\mu^{(str)}$ is reached on the boundary of the bubble. In the agreement with (26) and (3) we have the vector relation

$$\alpha_\mu^{(str)} = \alpha_\mu(r_b) = 2m/e, \tag{38}$$

which results in two very essential consequences [6]:

- the Higgs field (matched with the regularized KN electromagnetic field) forms a coherent vacuum state oscillating with the frequency $\omega = 2m$, which is a typical feature the "oscillon" soliton models.
- the regularized KN electromagnetic potential $\alpha_\mu^{(str)}$ forms on the boundary of the bubble a closed Aharonov-Bohm-Wilson quantum loop $\oint e\alpha_\phi^{(str)} d\phi = -4\pi ma$, which determines quantized spin of the soliton, $J = ma = n/2,\ n = 1, 2, 3, ...$

Does the KN model of electron contradict to Quantum Theory? It seems "yes", if one speaks on the "bare" electron. However, in accordance with QED, vacuum polarization creates in the Compton region a cloud of virtual particles forming a "dressed" electron. This region gives

contribution to electron spin, and performs a procedure of renormalization, which determines physical values of the electron charge and mass. Therefore, speaking on the "dressed" electron, one can say that the real contradiction between the KN model and the Quantum electron is absent.

Note that dynamics of the virtual particles in QED is chaotic and can be conventionally separated from the "bare" electron. In the same time, the vacuum state inside the Kerr-Newman soliton forms a *coherent oscillating state* joined with a closed Kerr string. It represents an *integral whole of the extended electron,* its 'internal' structure which cannot be separated from a "bare" particle. In any case, the Kerr string appears as an analogue of the pointlike bare electron.

6. Conclusion

We have showed that gravity definitely indicates presence of a closed string of the Compton radius $a = \hbar/(2m)$ in the electron background geometry. This string has gravitational origin and is close related with the fundamental closed strings of the low energy string theory. Corpuscular aspect of the traveling waves along the Kerr string allows us to 'derive' the Dirac equation. The original Dirac theory is modified in this case: the wave functions are factorized and acquire the singular stringlike carriers. As a result, the new wave functions turn out to be propagating along the 'axial' singular strings, which is reminiscent of the de Broglie wave-pilot conjecture. Therefore, the gravitational KN closed string represents a bridge between gravity, superstring theory and the Dirac quantum theory towards the consistency of these theories. We arrive at the extremely unexpected conclusion that Gravity, as a basic part of the superstring theory, may lie beyond Quantum theory and play a fundamental role in its 'emergence'.

The observable parameters of the electron determine unambiguously the Compton size of the Kerr string. This size is very big with respect to the modern scale of the experimental resolutions, and it seems, that this string should be experimentally detected. However, the high-energy scattering detects the pointlike electron structure down to $10^{-16}cm$. One of the explanations of this fact, given in [7], is related with the assumption that interaction of the KN particles occurs via the lightlike KN 'axial' strings. Just such a type of the 'direct' lightlike interaction follows from the analysis of the Kerr theorem for multiparticle Kerr-Schild solutions, [34]. So far as the KN circular string is also lightlike, the lightlike photon can contact it only at one point. The resulting scattering of the Kerr string by the *real* photons of high energy can exhibit only the pointlike interaction, and neither form of the string, nor its extension cannot be recognized. To recognize the shape of the string as a whole, it is necessary two extra conditions:

a) a *relative low-energy* resonance scattering with the wavelengths comparable with extension of the string. It means that there must be a scattering with a low-energy momentum transfer, i.e. with a small Bjorken parameter $x = q_t/P$.

b) simultaneously, to avoid the pointlike contact interaction, the scattering should be deeply virtual, i.e. the square of the transverse four-momentum transfer should satisfy $Q^2 >> m^2$.

Both these conditions correspond to the novel tool – the Deeply Virtual Compton Scattering (DVCS) described by the theory of Generalized Parton Distribution (GPD) [25, 26], which allows one to probe the shape of the elementary particles by the "non-forward Compton scattering" [27]. If the predicted Kerr-Newman string will be experimentally recognized in the core of electron structure, it could be great step in understanding Quantum theory towards to Quantum Gravity.

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